CEMC GRADES 9/10 MATH CIRCLES
NOVEMBER 23/30, 2022
FORMAL LOGIC - PROBLEM SET

1. Introduction

Find the error in the following arguments.

(1) I have only ever seen black cats. Therefore all cats must be black.
(2) Sharks eat fish. I am not a fish. Therefore a shark will not eat me.
(3) If I eat healthy and exercise I will become stronger. I eat healthy. Therefore I will become stronger.
(4) All stock brokers want to make money. Smarter investors will make more money. Money does not buy happiness. Therefore smart stock brokers will not be happy.

2. Implication

Transform the following statements into an implication using "If... then...".

(1) You'll catch a cold without a coat!
(2) Buy one get one free.
(3) I get sleepy when I read.
(4) Beautiful sunsets deserve to be seen.

What is the hypothesis and conclusion in the following implications?

(1) If he stands too close to the edge then he will fall.
(2) If everyone wants to go to the mall then we can go.

3. Other Logical Connectives

Let $P =$"I sing loudly", $Q =$"I dance well" $R =$"The audience claps". Write the following logical statements in English.

(1) $P \wedge R$

(2) $P \Rightarrow R$

(3) $(P \vee Q) \Rightarrow R$

(4) $(\neg R \Rightarrow \neg Q)$

Fill in the following truth tables

(1)

P	$P \vee \neg P$
T	
F	

(2)

Q	$Q \wedge \neg Q$
T	
F	

(3)

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$
T	T	T	T	
T	T	F		T
T	F	T	F	
T	F	F		T
F	T	T	T	
F	T	F		F
F	F	T	F	
F	F	F		F

(4)

P	$\neg P$	$\neg\neg P$
T	F	
F	T	

4. Tautologies and how to win arguments

Blank truth tables are provided below to help with proving.

Are the following formulas tautologies?

(1) $P \Rightarrow (P \wedge Q)$
(2) $\neg\neg P \Rightarrow P$
(3) $(P \vee \neg P) \Rightarrow Q$
(4) $(P \wedge \neg P) \Rightarrow Q$

Check if the following arguments are valid or not.

(1) If you give a mouse a cookie then he's going to want some milk.
(2) If you give a mouse some milk he's going to want a straw.
(3) Therefore if you give a mouse a cookie he's going to want a straw.

(1) If you're still doing these problems then you must like math
(2) Therefore if you do not like math you are not still doing these problems

(1) Every person is tall or short
(2) Every short person is rich or poor
(3) Therefore every person is tall or rich or poor

5. Semantics

Which of the following logical equivalences are correct?

(1) $(P \vee \neg P) \vee Q \equiv Q$

(2) $(P \wedge \neg P) \vee Q \equiv Q$

(3) $(P \vee \neg P) \wedge Q \equiv Q$

(4) $(P \wedge \neg P) \wedge Q \equiv Q$

Notice that $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$, and so we can write it without brackets as their is no confusion in saying $P \vee Q \vee R$.

WITHOUT using a truth table show that $\neg(P \vee Q \vee R) \equiv \neg P \wedge \neg Q \wedge \neg R$.

6. Contrapositive

Prove the following statements by contrapositive.

(1) If $x = 2$ then $3x - 5 \neq 10$.

(2) If x, y are two integers such that $x + y$ is even, then x and y have the same parity (both odd or both even).

7. Reductio ad absurdum

Prove the following statements by contradiction.

(1) If x^2 is even then x is even.

(2) If $x, y \in \mathbb{R}$ are such that $x > 0$ and $y > 0$ then if $xy > 25$ at least one of x, y must be greater than 5.

(3) The square root of an irrational number is irrational.